Quadratic programming ﻿﻿is a form of non-linear programming. The objective function contains at least one squared variable, such as: ax2 + bx +c = 0. the constraints are linear, whether they are equalities or inequalities. An example of quadratic programming is the portfolio optimization problem. In this example, we have 4 assets with total investment of $10,000. The covariance matrix of asset returns is shown as below. The expected value or return is [0.05, -0.2, 0.15, 0.30]. What would be the best allocation of each asset when we are expecting growth to be at least 10%.

A picture containing photo, meter

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Based on the matrix above, the quadratic equation would be written as:

Objective:

Constraints:

X1 + x2 + x3 + x4 <= $10,000

0.05x1 – 0.2x2 + 0.15x3 + 0.30x4 >= $1,000

X1, x2, x3, x4 >= 0

Therefore,

Z: investment risk, the variance of the portfolio’s total return

X: investment amount of each asset

Q =

C = 0

A =

b =

after calculation, the formulation has been solved as below,

x1 = 3,452, x2 = 0, x3 = 1,068, x4 = 2; 223

The Markowitz Portfolio Theory, also known as KPT, is an optimization model in finance to calculate the minimum risk of a portfolio returns with certain level of growth. The example that mentioned early is actually solved by KPT. It is a quadratic model when Ct equals to zero. The decision variables are the investment amount of each asset. The objective function is:

Minimum Variance of Portfolio = A close up of a logo

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Where s is the sample variance in the return of asset i, or covariance between assets i and j. The constraints would be the investment limits, expected level of returns, and the nonnegativity assumption. If the decision variables are all positive, it means that the short sells are not allowed, vice versa.

Option Pricing and Volatility models are the quadratic models that calculate the theoretical value of an option, as known as the fair value. The assumption of the model is all the price are lognormal distributed. The decision variables are the price of underlying asset, strike price, volatility, risk-free interest rate, and time to maturity. The objective function is shown as below. The constraints are volatility and the risk-free interest rate are assumed to be constant.

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Reference:

1

2

3